THE 1995 ASIAN PACIFIC MATHEMATICAL OLYMPIAD

Time allowed: 4 hours
NO calculators are to be used.
Each question is worth seven points.

Question 1

Determine all sequences of real numbers $a_1, a_2, \ldots, a_{1995}$ which satisfy:

$$2\sqrt{a_n - (n-1)} \geq a_{n+1} - (n-1), \text{ for } n = 1, 2, \ldots 1994,$$

and

$$2\sqrt{a_{1995} - 1994} \geq a_1 + 1.$$

Question 2

Let $a_1, a_2, \ldots, a_n$ be a sequence of integers with values between 2 and 1995 such that:

(i) Any two of the a_i's are realtively prime,
(ii) Each a_i is either a prime or a product of primes.

Determine the smallest possible values of n to make sure that the sequence will contain a prime number.

Question 3

Let $PQRS$ be a cyclic quadrilateral such that the segments PQ and RS are not parallel. Consider the set of circles through P and Q, and the set of circles through R and S. Determine the set A of points of tangency of circles in these two sets.

Question 4

Let C be a circle with radius R and centre O, and S a fixed point in the interior of C. Let AA' and BB' be perpendicular chords through S. Consider the rectangles $SAMB$, $SBN'A'$, $SA'M'B'$, and $SB'NA$. Find the set of all points M, N', M', and N when A moves around the whole circle.

Question 5

Find the minimum positive integer k such that there exists a function f from the set $\mathbb{Z}$ of all integers to $\{1, 2, \ldots k\}$ with the property that $f(x) \neq f(y)$ whenever $|x - y| \in \{5, 7, 12\}$.