

11th Asian Pacific Mathematical Olympiad

March, 1999

1. Find the smallest positive integer n with the following property: there does not exist an arithmetic progression of 1999 real numbers containing exactly n integers.
2. Let a_1, a_2, \dots be a sequence of real numbers satisfying $a_{i+j} \leq a_i + a_j$ for all $i, j = 1, 2, \dots$. Prove that

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \dots + \frac{a_n}{n} \geq a_n$$

for each positive integer n .

3. Let Γ_1 and Γ_2 be two circles intersecting at P and Q . The common tangent, closer to P , of Γ_1 and Γ_2 touches Γ_1 at A and Γ_2 at B . The tangent of Γ_1 at P meets Γ_2 at C , which is different from P , and the extension of AP meets BC at R . Prove that the circumcircle of triangle PQR is tangent to BP and BR .
4. Determine all pairs (a, b) of integers with the property that the numbers $a^2 + 4b$ and $b^2 + 4a$ are both perfect squares.
5. Let S be a set of $2n + 1$ points in the plane such that no three are collinear and no four concyclic. A circle will be called *good* if it has 3 points of S on its circumference, $n - 1$ points in its interior and $n - 1$ points in its exterior. Prove that the number of good circles has the same parity as n .