42^{nd} International Mathematical Olympiad

Washington, D.C., United States of America

Day I 9 a.m. - 1:30 p.m.

July 8, 2001

1. Let ABC be an acute-angled triangle with O as its circumcenter. Let P on line BC be the foot of the altitude from A. Assume that $\angle BCA \geq \angle ABC + 30^\circ$. Prove that $\angle CAB + \angle COP < 90^\circ$.

2. Prove that
$$\frac{a}{\sqrt{a^2+8bc}} + \frac{b}{\sqrt{b^2+8ca}} + \frac{c}{\sqrt{c^2+8ab}} \geq 1$$
for all positive real numbers a, b, and c.

3. Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that

   (a) each contestant solved at most six problems, and

   (b) for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy.

   Prove that there is a problem that was solved by at least three girls and at least three boys.

42^{nd} International Mathematical Olympiad

Washington, D.C., United States of America

Day II 9 a.m. - 1:30 p.m.

July 9, 2001

4. Let n be an odd integer greater than 1 and let $c_1, c_2, \ldots, c_n$ be integers. For each permutation $a = (a_1, a_2, \ldots, a_n)$ of $\{1, 2, \ldots, n\}$, define $S(a) = \sum_{i=1}^{n} c_i a_i$. Prove that there exist permutations b and c, $b \neq c$, such that $n!$ divides $S(b) - S(c)$.

5. In a triangle ABC, let segment AP bisect $\angle BAC$, with P on side BC, and let segment BQ bisect $\angle ABC$, with Q on side CA. It is known that $\angle BAC = 60°$ and that $AB + BP = AQ + QB$. What are the possible angles of triangle ABC?

6. Let $a > b > c > d$ be positive integers and suppose

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $ab + cd$ is not prime.